Advanced Pre-calculus Final Exam Review Fall Semester

1. Given: $f(x)=3x^2-4x+2$, *find* $f(-2)$. $f(-2)=3(-2)^2-4(-2)+2=\boxed{22}$

2. Find the distance between (3,-4) and (-1,-6). $\sqrt{(3-^-1)^2+(-4-^-6)^2}=\sqrt{20}=\boxed{2\sqrt{5}}\approx\boxed{4.47}$

3. Find the midpoint between (4,-3) and (-2,-7). $\left(\frac{4+^-2}{2},\frac{^-3+^-7}{2}\right)=\boxed{(1,^-5)}$

4. Find the equation of the line that passes through (4,-1) and (7,-3).
$m=\frac{^-3-^-1}{7-4}=\frac{^-2}{3}$ $b=-1-\frac{^-2}{3}\cdot 4=\frac{-5}{3}$ $\boxed{y=-\frac{2}{3}x+\frac{5}{3}}$

5. Find the equation of the line that passes through (3,-5) and is perpendicular to 2x-3y=5. $y=\frac{2}{3}x-\frac{5}{3}$
$m=\frac{^-3}{2}$ $b=-5-\frac{^-3}{2}\cdot 3=\frac{^-1}{2}$ $\boxed{y=\frac{^-3}{2}x-\frac{1}{2}}$

6. Determine whether the equation $x+y^2=4$ has y-axis, x-axis, or origin symmetry.
$(^-x)+y^2=4$ X $x+(-y)^2=4$ ✓ $(-x)+(-y)^2=4$ X $\boxed{\text{X-axis Symmetry}}$

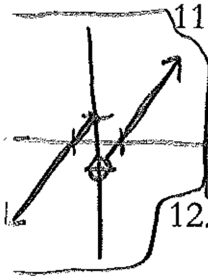
7. Given: $f(x)=2\sqrt{x-3}-1$, *find* $f^{-1}(x)$ *and state the domain.*
$x=2\sqrt{y-3}-1$ $x+1=2\sqrt{y-3}$ $\frac{x+1}{2}=\sqrt{y-3}$ $\boxed{\frac{(x+1)^2}{4}+3=y,\ x\geq 0}$

8. Given: $3-\sqrt{x^2-9}$, *find the domain and range.*
$\boxed{D:(-\infty,-3]\cup[3,\infty),\ R:(-\infty,3]}$

9. Given: $f(x)=x\sqrt{x+3}$, determine the interval(s) where the function is increasing.
$\boxed{(-2,\infty)}$

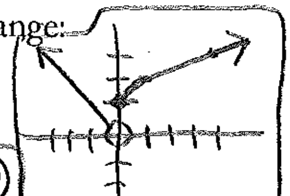
10. Given: $f(x)=x^2-2x+8$, find the average rate of change over the interval [1,5].
$f(1)=7,\ f(5)=23$ $m=\frac{23-7}{5-1}=\frac{16}{4}=\boxed{4}$

11. Sketch the graph of each piecewise function & state its domain and range:



a) $f(x)=\begin{cases}x+1, & x\leq 0\\ x-1, & x>0\end{cases}$ $\boxed{D:(-\infty,\infty)\ R:(-\infty,\infty)}$

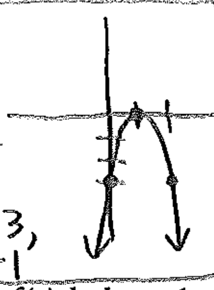
b) $f(x)=\begin{cases}|x|, & x<0\\ \sqrt{x}+1, & x\geq 0\end{cases}$ $\boxed{D:(-\infty,\infty)\ R:(0,\infty)}$



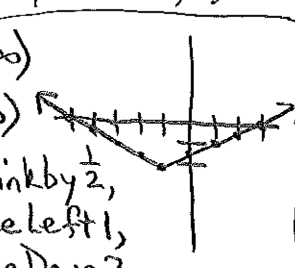
12. State the transformations, domain, range and sketch the graph of each:

a) $y=-3(x-1)^2$ b) $y=\frac{1}{2}|x+1|-2$ c) $y=-3\sqrt{-x-1}+3$

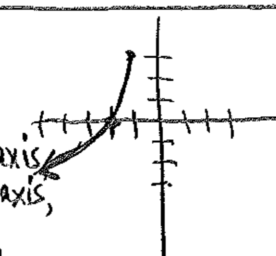
a) D: $(-\infty,\infty)$
R: $(-\infty,0]$
T: Reflection over x-axis, V. Stretch by 3, H. Slide Right 1



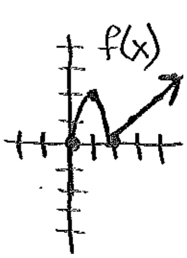
b) D: $(-\infty,\infty)$
R: $[-2,\infty)$
T: V. Shrink by $\frac{1}{2}$, H. Slide Left 1, V. Slide Down 2



c) D: $(-\infty,-1]$
R: $(-\infty,3]$
T: Reflect. over x-axis, Reflect. over y-axis, V. Stretch by 3, H. Slide Left 1, V. Slide Up 3



13. Given f(x) below sketch the graph of g(x) using transformations:

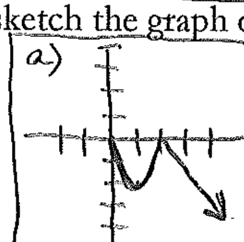


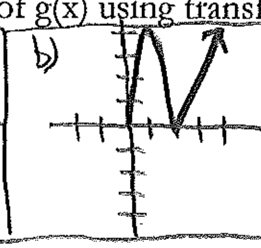
a) g(x) = -f(x)

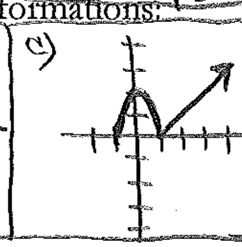
b) g(x)=2f(x)

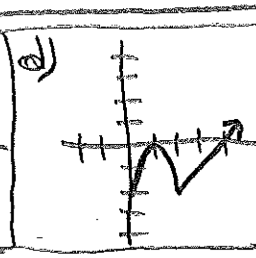
c) g(x)=f(x+1)

d) g(x)=f(x)-2

a)


b)


c)


d)


14. Given: $f(x)=\frac{1}{x}$, $g(x)=\sqrt{x+3}$, $h(x)=2x+1$, *find* : $f(2\sqrt{x+3}+1)=\boxed{\frac{1}{2\sqrt{x+3}+1}, x\geq -3}$

a) g(h(x)) and its domain $\boxed{\sqrt{2x+4}, x\geq -2}$

b) (f+h)(x) and its domain $\boxed{\frac{1}{x}+2x+1, x\neq 0}$

c) $\left(\frac{f}{g}\right)(x)$ and its domain $\boxed{\frac{1}{x\sqrt{x+3}}, -3<x<0 \cup x>0}$

d) f(h(g(x))) and its domain

e) State the inverse of g(x) and its domain

$x=\sqrt{y+3}$ $\boxed{g^{-1}(x)=x^2-3, x\geq 0}$

$x^2=y+3$

$x^2-3=y$

15. State two angles, one positive and one negative that are conterminal with 208°.

$\boxed{568° \qquad -152°}$

208+360 208-360

16. Find θ when the terminal side of θ passes through (-2,-2). Also find the 6 trig. ratio values of θ.

$\boxed{\theta=225°=\frac{5\pi}{4}}$ $\boxed{\sin\theta=-\frac{1}{\sqrt{2}} \quad \tan\theta=1 \quad \csc\theta=-\sqrt{2} \quad \cos\theta=-\frac{1}{\sqrt{2}} \quad \cot\theta=1 \quad \sec\theta=-\sqrt{2}}$

17. Name the quadrant where $\sin x<0$ and $\cot x>0$. $\boxed{\text{III}}$

18. Express each trig ratio in terms of its reference angle;

a) $\cos(420°)=\boxed{\cos 60°}=\boxed{\frac{1}{2}}$ b) $\sin(-150°)=\boxed{-\sin 30°}=\boxed{-\frac{1}{2}}$

c) $\tan(662°)=\tan 302°=\boxed{-\tan 58°}$ d) $\sec\left(\frac{8\pi}{3}\right)=\sec\frac{2\pi}{3}=\boxed{-\sec\frac{\pi}{3}}=\boxed{-2}$

19. Complete the chart: (no decimals)

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
a. 120°	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$2/\sqrt{3}$	-2	$-1/\sqrt{3}$
b. 330°	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$	-2	$2/\sqrt{3}$	$-\sqrt{3}$
c. 270°	-1	0	Undef.	-1	Undef.	0

20. Given: $\cos\theta=\left(-\frac{1}{2}\right)$ and $\tan\theta=\sqrt{3}$, What quadrant does θ belong $\boxed{\text{III}}$,

21. Find θ from problem #20 θ= $\boxed{240°}=\boxed{\frac{4\pi}{3}}$

22. Change into radians (leave in terms of π)

a) 30° $\boxed{\frac{\pi}{6}}$ b) 120° $\boxed{\frac{2\pi}{3}}$ c) -240° $\boxed{-\frac{4\pi}{3}}$ d) -800° $\boxed{-\frac{40\pi}{9}}$

23. Change to degrees:

a) $\left(\frac{2\pi}{3}\right)$ $\boxed{120°}$ b) $\left(\frac{-3\pi}{2}\right)$ $\boxed{-270°}$ c) (14π) $\boxed{2520°}$ d) $\left(\frac{7\pi}{4}\right)$ $\boxed{315°}$

24. Determine the radian measure of the central angle of a circle of radius 210 miles that intercepts an arc length of 375 miles.

$\ell = r\theta \qquad \theta = \frac{\ell}{r} = \frac{375}{210} = \boxed{\frac{25}{14}}$

25. Determine the arc length of a circle with radius 67 cm and central angle intercepting the arc of 250°.

$\ell = r\theta = 67 \cdot \frac{250\pi}{180} = \boxed{\frac{1675\pi}{18}\text{ cm}} \approx \boxed{292.34\text{ cm}}$

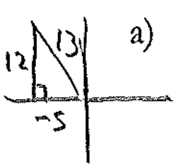
26. Determine the area of the sector of a circle with radius of 22.75 inches and a central angle of 122°.

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(22.75)^2 \cdot \frac{122\pi}{180} \approx \boxed{551.02\text{ in}^2}$

27. State three consecutive asymptotes of the function: $y = -3\csc 3x + 2$ (state the domain)

$x = \frac{n\pi}{3} \qquad \boxed{0, \frac{\pi}{3}, \frac{2\pi}{3}}$

28. If $\cos\beta = \left(-\frac{5}{13}\right)$. Where β is in quadrant II, find:



a) $\sin\beta$ $\boxed{\frac{12}{13}}$ b) $\tan\beta$ $\boxed{-\frac{12}{5}}$ c) $\csc\beta$ $\boxed{\frac{13}{12}}$ d) $\sec\beta$ $\boxed{-\frac{13}{5}}$ e) $\cot\beta$ $\boxed{-\frac{5}{12}}$

29. Given the equation $y = a\cos b(x - c) + d$ tell the transformations which a, b, c, and d could change the equation $y = \cos x$

a: V. Stretch/V. Shrink, Reflection over x-axis
b: H. Stretch/H. Shrink, Reflection over y-axis
c: H. Slide
d: V. Slide

30. Given the equation $y = \left(\frac{1}{2}\right)\csc(2x - \pi) - 3$: state the period, range values, domain values, phase shift, and transformations on the curve $y = \csc x$

$(2(x - \frac{\pi}{2}))$

$\frac{n\pi}{2} + \frac{\pi}{2}$

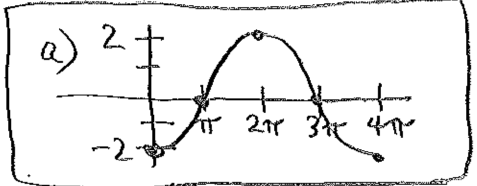
Period $= \pi$
Range: $(-\infty, -3\frac{1}{2}] \cup [-2\frac{1}{2}, \infty)$
Domain: $\{x \mid x \neq \frac{n\pi}{2} + \frac{\pi}{2}\}$
Phase shift: Right $\frac{\pi}{2}$
V. Shrink by $\frac{1}{2}$
H. Shrink by 2
V. Shift Down 3

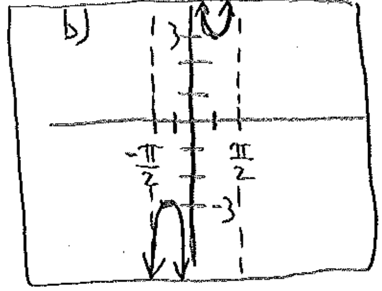
31. Given the equation $y = -3\tan\left(x - \left(\frac{\pi}{2}\right)\right) + 2$: State the period, domain values, range values, phase shift, and transformations on the curve $y = \tan x$

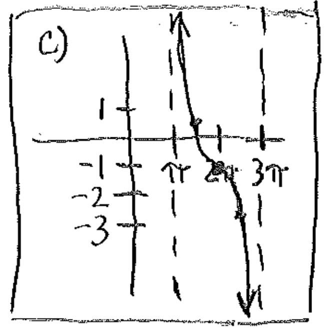
$\frac{\pi}{2} + n\pi + \frac{\pi}{2}$

Period $= \pi$
Domain: $\{x \mid x \neq \pi + n\pi\}$
Range: $(-\infty, \infty)$
Phase shift Right $\frac{\pi}{2}$
Reflection over x-axis
V. Stretch by 3
V. Shift Up 2

32. Graph:







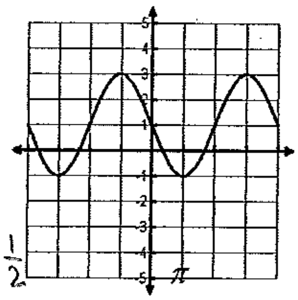
a) $y = -2\cos\left(\frac{1}{2}\right)x$

b) $y = -3\csc(2x + \pi)$
$(2(x+\frac{\pi}{2}))$

c) $y = \left(\frac{3}{2}\right)\cot\left(\frac{1}{2}\right)(x - \pi) - 1$

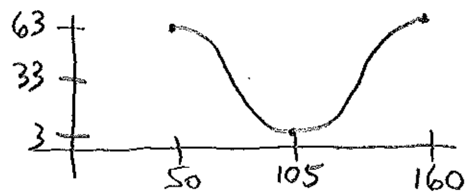
33. Write the equation of the graph below: (no phase shift)

$\underline{-\sin}$
$c=0$
$d=1$
$a=2$
$b=\frac{2\pi}{4\pi}=\frac{1}{2}$



$y = \boxed{-2\sin\left(\frac{1}{2}x\right)+1}$

34. As you ride the Ferris wheel at the park, your distance from the ground varies sinusoidally with time. You start a stopwatch, when the time reads 50 seconds, your seat is at the highest point on the wheel. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 110 seconds to make a complete revolution. The diameter of the wheel is 60 feet and sits 3 feet above the ground.



$\underline{\cos}$
$c=50$
$d=33$
$a=30$
$b=\frac{2\pi}{110}=\frac{\pi}{55}$

$y=\boxed{30\cdot\cos\left(\frac{\pi}{55}(x-50)\right)+33}$ a. Write an equation.

$\boxed{73.38 \text{ seconds}}$ b. Determine the time you will be 40 feet above the ground for the second time.

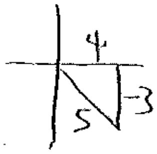
35. Evaluate (give as exact values):

a) $\sin^{-1}\left(\frac{1}{2}\right)$ $\boxed{30^\circ = \frac{\pi}{6}}$

b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\boxed{150^\circ = \frac{5\pi}{6}}$

c) $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$ $\boxed{-\frac{1}{\sqrt{3}}}$

d) $\sec\left(\sin^{-1}\left(\frac{12}{13}\right)\right)$ $\boxed{\frac{13}{5}}$



e) $\cos\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)$ $\boxed{\frac{4}{5}}$

f) $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$ $\boxed{45^\circ = \frac{\pi}{4}}$
$\tan^{-1}(1)$

36. If $x = \frac{\pi}{4}$, then $\sin x$ $\boxed{=}$ $\cos x$ (>,<, or =)

37. Given triangle ABC where $\angle C = 90°, \angle A = 22°13', b = 15cm$. Find c to the nearest thousandth of a cm.

$\angle B = 180 - 90 - 22°13' = 67°47'$ $\quad \frac{\sin 67°47'}{15} = \frac{\sin 90}{c}$

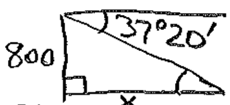
$c = 15 \cdot \sin 90° / \sin 67°47' = \boxed{16.203 \text{ cm}}$

38. Given triangle ABC where $\angle C = 90°, a = 11.4$ and $c = 32.4$, find $\angle$A to the nearest tenth of a degree.

$\frac{\sin 90°}{32.4} = \frac{\sin A}{11.4}$ $\quad \sin A = \frac{11.4 \cdot \sin 90°}{32.4}$

$\angle A = \sin^{-1}(\text{Ans}) = \boxed{20.6°}$

39. The angle of depression from the top of a cliff 800 meters high to the base of a log cabin is 37°20'. How far is the cabin from the foot of the cliff to the nearest tenth of a meter?



$\tan 37°20' = \frac{800}{x}$ $\quad x = \frac{800}{\tan 37°20'} = \boxed{1048.9 \text{ m}}$

40. Given triangle ABC, find all values to the nearest hundredth:

a) b, when $\angle$A=51°, $\angle$C=67°, and a = 8

$\angle B = 180 - 51 - 67 = 62°$

$\frac{\sin 51°}{8} = \frac{\sin 62°}{b}$ $\quad b = \frac{8 \cdot \sin 62°}{\sin 51°} = \boxed{9.09}$

b) c, when $\angle$A=43°, a = 7, and b = 4

$\frac{\sin 43°}{7} = \frac{\sin B}{4}$ $\quad \angle B = \sin^{-1}(4 \cdot \sin 43°/7) = \underline{22.94°}$ $\quad \angle C = 180 - 43 - B = \underline{114.06°}$ $\quad \frac{\sin 43°}{7} = \frac{\sin C}{c}$ $\quad c = \frac{7 \cdot \sin C}{\sin 43°}$

$\boxed{c = 9.37}$

c) Find $\angle$C if $\angle$A=40°, a = 9, and b = 10

$\frac{\sin 40°}{9} = \frac{\sin B}{10}$ $\quad \angle B = \sin^{-1}(10 \cdot \sin 40°/9) = \underline{45.58}$ $\quad \angle C = 180 - 40 - B = \boxed{94.42°}$

d) Area of a triangle when $a = 10, b = 16, \angle C = 30°$

$A_\triangle = \frac{1}{2} \cdot 10 \cdot 16 \cdot \sin 30° = \boxed{40}$

e) Area of a triangle when $a = 10, b = 14, c = 16$

$S = \frac{10+14+16}{2} = 20$ $\quad A_\triangle = \sqrt{20(20-10)(20-14)(20-16)} = \boxed{69.28}$

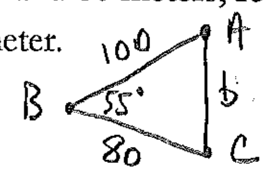
f) Solve: ΔLAW, given $\angle L$=48°, a=20.4 cm, and w=12.6 cm.

$\angle A = 180 - 48 - W = \boxed{93.96°}$

$l = \sqrt{20.4^2 + 12.6^2 - 2 \cdot 20.4 \cdot 12.6 \cdot \cos 48°} = \boxed{15.20 \text{ cm} = l}$

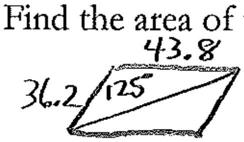
$\frac{\sin 48°}{l} = \frac{\sin W}{12.6}$ $\quad \angle W = \sin^{-1}\left(\frac{12.6 \cdot \sin 48°}{l}\right) = \boxed{38.04°}$

41. The distance from a boat at point B to point A and C on the shore are known to be 100 meters and 80 meters, respectively, and $\angle$ABC=55°. Find AC to the nearest hundredth of a meter.



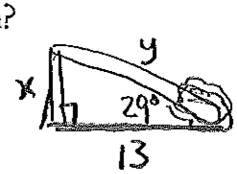
$b = \sqrt{80^2 + 100^2 - 2 \cdot 80 \cdot 100 \cdot \cos 55°} = \boxed{84.99 \text{ m}}$

42. The sides of a parallelogram are 43.8 cm and 36.2 cm, and the measure of one angle is 125°. Find the area of the parallelogram



$A_{\square} = 2 \cdot A_\triangle = 2 \cdot \frac{1}{2} \cdot 36.2 \cdot 43.8 \cdot \sin 125° = \boxed{1298.81 \text{ cm}^2}$

43. A tree is broken by the wind. The top touches the ground 13 meters from the base and makes an angle with the ground measuring 29°. How tall was the tree before it was broken?

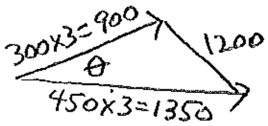


$\tan 29° = \frac{x}{13}$ $\quad \cos 29° = \frac{13}{y}$

$13 \cdot \tan 29° = x = \underline{7.21}$ $\quad y = \frac{13}{\cos 29°} = \underline{14.86}$

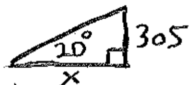
$x + y = \boxed{22.07 \text{ m}}$

44. Two planes, one flying 300 mph and the other 450 mph, left an airport at the same time. Three hours later they were 1200 miles apart. What was the measure of the angle between their flight paths?



$\theta = \cos^{-1}\left(\frac{900^2+1350^2-1200^2}{2\cdot 900\cdot 1350}\right) = \boxed{60.61^\circ}$

45. A ship is just offshore of New York City. A sighting is taken of the Statue of Liberty, which is about 305 feet tall. If the angle of elevation to the top of the statue is 20°, how far is the ship from the base of the statue?



$\tan 20^\circ = \frac{305}{x}$ $\quad x = \frac{305}{\tan 20^\circ} = \boxed{837.98 \text{ ft}}$

46. Determine the area of a triangular lot if the sides measure 120 ft, 250 ft, and 150ft.

$s = \frac{120+250+150}{2} = \frac{520}{2} = 260$ $\quad A_\Delta = \sqrt{260(260-120)(260-250)(260-150)}$

$= \boxed{6327.72 \text{ ft}^2}$

Additional Textbook Practice

Chapter 4:

Pages 365-368 Review Exercises

35, 39, 43, 47, 53, 55, 57, 69, 77, 81, 87, 91, 95, 97 (a),109, 115-121 odd,

127-130, 133-135, 141-144

Page 369 Chapter Test: Problems: 1, 3, 4-8, 10-12, 17, 19, 20

~~**Chapter 5:**~~ (OMIT)

~~Pages 420 Review Exercises Sections: 5.1, 5.2, 5.3~~

Chapter 6:

Pages 482-483: 1-36 Sections 6.1, 6.2

Page 486 Chapter Test: Problems: 1-8