Advanced Precalculus
Review over Chapter 3

Name _____
Date _____ **Period** _____

I. Simplify:

1) $x^{\frac{1}{3}} \cdot x^{\frac{3}{2}} = x^{\left(\frac{1}{3}+\frac{3}{2}\right)} = \boxed{x^{\frac{11}{6}}}$

2) $2x(49)^0 = 2x \cdot 1 = \boxed{2x}$

3) $\dfrac{(27)^{-2}}{(27)^{-\frac{2}{3}}} = 27^{\left(-2-\left(-\frac{2}{3}\right)\right)} = 27^{-\frac{4}{3}} = \boxed{\dfrac{1}{81}}$

4) $\dfrac{(4x)^2\,(2x)^3}{2x^{-4}} = \dfrac{16x^2 \cdot 8x^3}{2x^{-4}} = \boxed{64x^9}$

5) $\dfrac{8x^2+4x^6}{(2x)^{-2}} = \boxed{2x^4+x^8}$

6) $\left(3x^{-4}y\right)^3\left(-2xy\right)^2 = 108x^{-10}y^5 = \boxed{\dfrac{108y^5}{x^{10}}}$

$27x^{-12}y^3 \cdot 4x^2y^2$

$4x^{-2}$

7) $\left(x^{\frac{3}{2}}\,y^{\frac{1}{3}}\,z^{-\frac{1}{2}}\right)^6 = x^9y^2z^{-3} = \boxed{\dfrac{x^9y^2}{z^3}}$

8) $\left(4x^4\right)^{\frac{3}{2}}\left(x^{-2}+2x^3\right) = 8x^6\left(x^{-2}+2x^3\right) = \boxed{8x^4+16x^9}$

9) $\sqrt{x^2yz} \cdot \sqrt[3]{zx^4y^5} = xy^{\frac{1}{2}}z^{\frac{1}{2}} \cdot z^{\frac{1}{3}}x^{\frac{4}{3}}y^{\frac{5}{3}} = \boxed{x^{\frac{7}{3}}y^{\frac{13}{6}}z^{\frac{5}{6}}}$

10) $\dfrac{x^{-\frac{3}{4}}y^{-\frac{1}{6}}}{x^{-\frac{1}{3}}y^{-\frac{2}{3}}} = x^{\left(-\frac{3}{4}-\left(-\frac{1}{3}\right)\right)}y^{\left(-\frac{1}{6}-\left(-\frac{2}{3}\right)\right)} = x^{-\frac{5}{12}}y^{\frac{3}{6}} = \boxed{\dfrac{y^{\frac{1}{2}}}{x^{\frac{5}{12}}}}$

II Solve for x using the one-to-one property:

11) $\left(\frac{1}{3}\right)^x = 81$

$3^{-x} = 3^4$ → $-x = 4$

$\boxed{x = -4}$

12) $2^{x+1} = \frac{1}{16}$

$2^{x+1} = 2^{-4}$

$x+1 = -4$

$\boxed{x = -5}$

13) $\left(\frac{1}{64}\right)^{3x-1} = (4)^{x-1}$

$\left(4^{-3}\right)^{3x-1} = 4^{x-1}$

$-9x+3 = x-1$

$-10x = -4$

$x = \frac{4}{10} = \boxed{\frac{2}{5}}$

14) $\left(\left(x+2\right)^{\frac{1}{2}}\right)^2 = \left(\left(\frac{1}{8}\right)^{\frac{1}{3}}\right)^2$

$x+2 = \left(\frac{1}{8}\right)^{\frac{2}{3}}$

$x+2 = \frac{1}{4}$

$\boxed{x = -\frac{7}{4}}$

15) $x^{\frac{3}{2}} = 64$

$x = 64^{2/3}$

$\boxed{x = 16}$

16) $125 = x^{-\frac{3}{2}}$

$125^{-2/3} = x$

$\frac{1}{125^{2/3}} = x$

$\boxed{x = \frac{1}{25}}$

III. Solve.

17) $\log_2(x-3) = 4$

$2^4 = x-3$

$16 = x-3$

$\boxed{19 = x}$

18) $\log_3\left(x^2-8\right) = 0$

$3^0 = x^2-8$

$1 = x^2-8$

$9 = x^2$ $\boxed{x = \pm 3}$

19) $\log 9x + \log x = 4$

$\log\left(9x^2\right) = 4$

$10^4 = 9x^2$

$\frac{10000}{9} = x^2$ → $\pm\sqrt{\frac{10000}{9}} = x$

$\pm\frac{100}{3} = x$

$\boxed{\frac{100}{3} = x}$

20) $\log_8(x+1) - \log_8 x = \log_8 4$

$\log_8\left(\frac{x+1}{x}\right) = \log_8 4$ → $x+1 = 4x$

$\frac{x+1}{x} = 4$ $1 = 3x$

$\boxed{\frac{1}{3} = x}$

21) $e^x = 18$

$\ln 18 = x$

$\boxed{x = 2.890}$

22) $5^x = 4$

$\log_5 4 = x$

$\boxed{x = 0.861}$

23) $\frac{1}{3}\log_5 x + \frac{2}{3}\log_5 x = 3$

$\log_5 x^{\frac{1}{3}} + \log_5 x^{\frac{2}{3}} = 3$

$\log_5\left(x^{\frac{1}{3}} \cdot x^{\frac{2}{3}}\right) = 3$ → $\log_5 x = 3$

$5^3 = x$

$\boxed{125 = x}$

24) $\log_8(x-1) = \log_8(x-2) - \log_8(x+2)$

$\log_8(x-1) = \log_8\left(\frac{x-2}{x+2}\right)$ → $(x-1)(x+2) = x-2$

$x-1 = \frac{x-2}{x+2}$ $x^2+x-2 = x-2$

$x^2 = 0$

$x = 0$ $\boxed{\emptyset}$

25) $\log(1-x) = -1$

$10^{-1} = 1-x$

$\frac{1}{10} = 1-x$

$x = 1-\frac{1}{10}$ $\boxed{x = \frac{9}{10}}$

26) $5\left(10^{x-6}\right) = 7$

$10^{x-6} = \frac{7}{5}$

$\log_{10}\left(\frac{7}{5}\right) = x-6$

$\log\left(\frac{7}{5}\right)+6 = x$

$\boxed{x = 6.146}$

27) $\log_3(x+6) + \log_3(x-2) = 2$

$\log_3((x+6)(x-2)) = 2$

$3^2 = x^2+4x-12$

$9 = x^2+4x-12$ → $0 = x^2+4x-21$

$0 = (x+7)(x-3)$

$x = -7, 3$ $\boxed{x = 3}$

IV. Set up the following and solve.

28) In twelve days, 1000 grams of an element decays to 200 grams. What is the half-life of this element? Express your answer to 3 decimal places.

$A = P\left(\frac{1}{2}\right)^{t/h}$ $200 = 1000\left(\frac{1}{2}\right)^{12/h}$ $\frac{200}{1000} = \left(\frac{1}{2}\right)^{12/h}$ $\log_{(\frac{1}{2})}\left(\frac{2}{10}\right) = \frac{12}{h}$ $h = \frac{12}{\log_{(\frac{1}{2})}\left(\frac{2}{10}\right)} = 5.168$ days

29) A person invests \$10,000 in some real estate. The investment grows at the rate of 12% per year, and is compounded every 3 months. When will the investment be worth \$15,000? Round to 3 decimal places.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $15{,}000 = 10{,}000\left(1+\frac{.12}{4}\right)^{4\cdot t}$ $\frac{15000}{10000} = \left(1+\frac{.12}{4}\right)^{4t}$ $\log_{(1+\frac{.12}{4})}(1.5) = 4t$ $\frac{\log_{(1+\frac{.12}{4})}(1.5)}{4} = t = 3.429$ years

30) Suppose an \$18,000 car depreciates 30% each year. What will its value be in 3 years, to the nearest cent?

$A = P(1-r)^t$ $A = 18000(1-.30)^3 = \$6174$

31) The number of disk drives D a trainee can assemble per day after t days of training can be modeled by the equation $D = 50 - 50e^{-.09t}$. Determine the number of days of training a person would need to be able to assemble 18 disk drives per day. Round to 3 decimal places.

$18 = 50 - 50e^{-.09t}$ $-32 = -50e^{-.09t}$ $\frac{32}{50} = e^{-.09t}$ $\ln\left(\frac{32}{50}\right) = -.09t$ $\frac{\ln\left(\frac{32}{50}\right)}{-.09} = t = 4.959$ days

32) In a typing class, the average number N of words per minute typed after t weeks of lessons was found to be $N = \frac{157}{1+5.4e^{-0.12t}}$. Find the time necessary to type each of the following. Round to 2 decimals.

a) 50 words per minute $50 = \frac{157}{1+5.4e^{-0.12t}}$ $1+5.4e^{-.12t} = \frac{157}{50}$ $5.4e^{-.12t} = \frac{107}{50}$ $e^{-.12t} = \left(\frac{107}{50}\right)/5.4$ $\ln\left(\left(\frac{107}{50}\right)/5.4\right) = -.12t$ $\ln\left(\left(\frac{107}{50}\right)/5.4\right)/-.12 = t$ $t = 7.713$ wks

b) 75 words per minute $75 = \frac{157}{1+5.4e^{-.12t}}$ $1+5.4e^{-.12t} = \frac{157}{75}$ $5.4e^{-.12t} = \frac{82}{75}$ $e^{-.12t} = \left(\frac{82}{75}\right)/5.4$ $\ln\left(\left(\frac{82}{75}\right)/5.4\right)/-.12 = t = 13.310$ wks

33) A deposit of \$10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

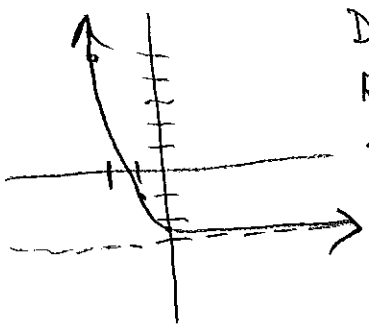
$A = P\cdot e^{rt}$

a) What is the annual interest rate for this account? $20{,}000 = 10{,}000\cdot e^{r\cdot 5}$ $2 = e^{5r} \rightarrow \ln 2 = 5r \rightarrow \frac{\ln(2)}{5} = r = .1386$ $r = 13.86\%$

b) Find the balance after 1 year. $A = 10{,}000\cdot e^{r\cdot 1} = \$11{,}486.98$

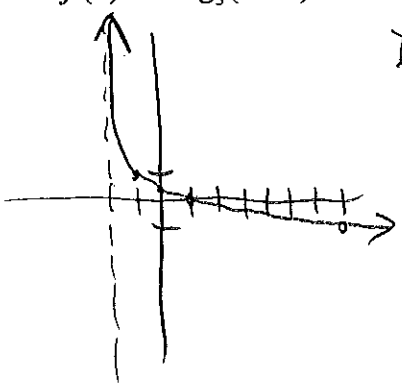
Sketch the graph.

34) $f(x) = 2\cdot 4^{-x-1} - 3$



D: $(-\infty,\infty)$
R: $(-3,\infty)$
x-int: $(-1.29, 0)$
y-int: $(0,-2.5)$
Asymptote: $y=-3$

35) $f(x) = -\log_3(x+2)+1$



D: $(-2,\infty)$
R: $(-\infty,\infty)$
x-int: $(1,0)$
y-int: $(0,0.369)$
Asymptote: $x=-2$