PRECALCULUS ADVANCED
Chapter 9 Review

1. Find the first five terms of the infinite sequence defined by $a_n = 3^n + 2$.

5, 11, 29, 83, 245

2. Find the first five terms of the sequence given by the recursive formula $a_1 = -1$ and $a_n = a_{n-1} + 5$.

-1, 4, 9, 14, 19

3. Write a recursive formula for the sequence 4, 12, 36, 108, ...

$a_1 = 4,\ a_{k+1} = a_k \cdot 3$

4. Find the 53rd term of the sequence 3, 11, 19, 27, 35, ...

$d = 8 \quad a_{53} = 3 + 8(52) = 419$

5. In an arithmetic sequence, $a_3 = 15$ and $a_8 = 50$. Find an explicit formula for the general term.

$d = \frac{50-15}{8-3} = \frac{35}{5} = 7 \quad a_1 = 15 - 7 \cdot 2 = 1 \quad a_n = 1 + 7(n-1)$

6. Find an explicit formula for the geometric sequence 120, 60, 30, 15, ..., and use it to find the eighth term.

$r = \frac{1}{2} \quad a_n = 120\left(\frac{1}{2}\right)^{n-1} \quad a_8 = 120\left(\frac{1}{2}\right)^7 = \frac{120}{128} = \frac{15}{16}$

7. Find four arithmetic means between 1 and −89.

$d = \frac{-89-1}{6-1} = \frac{-90}{5} = -18 \quad -17, -35, -53, -71$

8. Find three geometric means between 240 and 15.

$r = \sqrt[5-1]{\frac{15}{240}} = \sqrt[4]{\frac{1}{16}} = \pm\frac{1}{2} \quad \pm 120, 60, \pm 30$

9. Express $2 + 5 + 8 + 11 + \ldots$ using sigma notation.

$\sum_{n=1}^{\infty}(2+3(n-1))$

10. Express $2 + 5 + 8 + 11 + \ldots + 47$ using sigma notation.

$47 = 2 + 3(n-1) \Rightarrow 45 = 3(n-1) \Rightarrow 15 = n-1 \Rightarrow 16 = n$

$\sum_{n=1}^{16}(2+3(n-1))$

11. Find the sum of $\sum_{k=1}^{16}(3k-4)$.

$S_{16} = \frac{16}{2}(-1+44) = 8 \cdot 43 = 344$

12. Find $S_6$ for $\sum_{k=1}^{\infty}(10k-6)$.

$S_6 = \frac{6}{2}(4+54) = 3 \cdot 58 = 174$

13. Find $S_5$ for the geometric series with $a_1 = 36$ and $r = \frac{1}{3}$.

$S_5 = \frac{36\left(1-\left(\frac{1}{3}\right)^5\right)}{\left(1-\left(\frac{1}{3}\right)\right)} = 53.7 = \frac{484}{9}$

14. Find $S_{17}$ for $14 + 9 + 4 - 1 - \ldots$

$a_{17} = 14 - 5(16) = -66 \quad S_{17} = \frac{17}{2}(14-66) = \frac{17}{2} \cdot (-52) = -442$

15. Find the sum of $\sum_{k=1}^{5} k^4$.

$1 + 16 + 81 + 256 + 625 = 979$

16. The fourth term of a series is $17x$, and the sixth term is $23x$. Find the sum of the first twenty terms.

$d = \frac{23x-17x}{6-4} = \frac{6x}{2} = 3x \quad a_1 = 17x - 3(3x) = 8x$

$a_{20} = 8x + 3x(19) = 65x \quad S_{20} = \frac{20}{2}(8x+65x) = 10(73x) = 730x$

17. Find the sum of $7 + 4 + 1 - 2 - \ldots - 26$.

$d = -3 \quad -26 = 7 - 3(n-1) \Rightarrow -33 = -3(n-1) \Rightarrow 11 = n-1 \Rightarrow 12 = n$

$S_{12} = \frac{12}{2}(7-26) = 6(-19) = -114$

18. Find the sum of $96 + 48 + 24 + \ldots + \frac{3}{2}$.

$r = \frac{1}{2} \quad \frac{3}{2} = 96\left(\frac{1}{2}\right)^{n-1} \quad \frac{1}{64} = \left(\frac{1}{2}\right)^{n-1} \quad \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{n-1} \Rightarrow 6 = n-1 \Rightarrow 7 = n$

$S_7 = \frac{96\left(1-\left(\frac{1}{2}\right)^7\right)}{\left(1-\frac{1}{2}\right)} = 190.5$

19. Your Aunt Alice sends you $50 for your eighth birthday. On each birthday after that, she sends you $25 more. How much will she send you on your eighteenth birthday?

$a_1 = 50,\ d = 25 \quad a_{11} = 50 + 25(10) = \$300$

20. You interview for a job where the boss tells you he will pay you $0.01 the first day, $0.02 the second day, $0.04 the third day, $0.08 the fourth day, etc. If you take the job and start on June 1, how much would you make on the day of June 30?

$a_1 = .01 \quad r = 2 \quad a_{30} = \frac{.01(1-2^{30})}{1-2} = \$5,368,709.11$

21. Find the value of each of the following. Do not give answers as decimals.

(a) $\frac{76!}{77!} = \frac{1}{77}$ (b) $\frac{95!}{93!} = \frac{95 \cdot 94 \cdot 93!}{93!} = 8930$ (c) $\frac{n!}{(n+2)!} = \frac{1}{(n+2)(n+1)} = \frac{1}{n^2+3n+2}$

Find the sum.

22. $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots \quad r = \frac{1}{3},\ a_1 = \frac{2}{3} \quad S_\infty = \frac{\frac{2}{3}}{1-\frac{1}{3}} = 1$

23. $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \ldots \quad r = -\frac{1}{3},\ a_1 = \frac{2}{3} \quad S_\infty = \frac{\frac{2}{3}}{1-\left(-\frac{1}{3}\right)} = \frac{1}{2}$

24. $\sum_{k=1}^{\infty} 12\left(\frac{2}{3}\right)^k \quad r = \frac{2}{3},\ a_1 = 8 \quad S_\infty = \frac{8}{1-\frac{2}{3}} = 24$

25. $\sum_{k=1}^{\infty}\left(-\frac{5}{4}\right)^k$ Not Possible $(|r| > 1)$

Use the Binomial Theorem on problems 26 – 29.

26. Expand and simplify: $(3x-2y)^6$.

$(3x)^6 + 6(3x)^5(-2y)^1 + 15(3x)^4(-2y)^2 + 20(3x)^3(-2y)^3 + 15(3x)^2(-2y)^4 + \ldots$

$729x^6 - 1916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$

27. Expand and simplify: $(5x^2+y^3)^4$.

$(5x^2)^4 + 4(5x^2)^3(y^3)^1 + 6(5x^2)^2(y^3)^2 + 4(5x^2)^1(y^3)^3 + (y^3)^4$

$625x^8 + 500x^6y^3 + 150x^4y^6 + 20x^2y^9 + y^{12}$

28. Find the first three terms of $(a-b)^{20}$, and simplify.

$a^{20} + 20a^{19}(-b)^1 + {}_{20}C_2 \cdot a^{18}(-b)^2 = a^{20} - 20a^{19}b^1 + 190a^{18}b^2$

29. Find the 5th term of $(2x^2-3y)^{10}$

${}_{10}C_4 \cdot (2x^2)^6(-3y)^4 = 15,088,640x^{12}y^4$