Name _____
Date _____

Advanced Precalculus
2nd Semester Final Review

Chapter 2

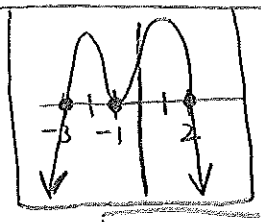
1) A person throws a ball straight up in the air. The height of the ball in feet, after t seconds, is given by the equation $h(t) = -16t^2 + 45t + 5$.
   a) Find the maximum height the ball reaches. 36.64 ft
   b) How much time elapses before the ball reaches the ground? 2.92 seconds
   c) At what time(s) will the ball reach a height of 10 feet? .12 seconds and 2.70 seconds
   d) How high is the ball after 2 seconds? 31 feet

2) Divide: $\frac{4x^6 - 3x^4 + 5x^2 - 2x + 3}{x+3}$

$-3 \mid 4\ 0\ -3\ 0\ 5\ -2\ 3$
$-12\ 36\ -99\ 297\ -906\ 2724$
$4\ -12\ 33\ -99\ 302\ -908\ 2727$

Quotient: $4x^5 - 12x^4 + 33x^3 - 99x^2 + 302x - 908$
Remainder 2727

3) Sketch the graph of $P(x) = -(x-2)(x+1)^2(x+3)$.



4) Find all the zeros of the polynomial $f(x) = x^3 - x^2 - 20x + 50$.

$-5 \mid 1\ -1\ -20\ 50$
$1\ -6\ 10\ 0$

$x^2 - 6x + 10$
$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$

$-5, 3 \pm i$

5) Find the value of k so that (x+2) is a factor of $f(x) = 2x^3 + 4x^2 + kx + 3$.

$-2 \mid 2\ 4\ k\ 3$
$-4\ 0\ -2k$
$2\ 0\ k\ -2k+3$

$-2k + 3 = 0$
$-2k = -3$
$k = \frac{3}{2}$

6) Given the following rational function $f(x) = \frac{3x-5}{x+4}$,
   a) find the x-intercepts. $(\frac{5}{3}, 0)$
   b) find the y- intercept. $(0, -\frac{5}{4})$
   c) state the equation of the horizontal asymptote. $y = 3$
   d) state the equation(s) of the vertical asymptote(s). $x = -4$

7) Given the following rational function $f(x) = \frac{x-1}{x^2+2x-3} = \frac{x-1}{(x+3)(x-1)} = \frac{1}{x+3}$
   a) find the x-intercepts. None
   b) find the y- intercept. $(0, \frac{1}{3})$
   c) state the equation of the horizontal asymptote. $y = 0$
   d) state the equation(s) of the vertical asymptote(s). $x = -3$
   e) state the domain of the function. $\{x \mid x \neq 1, -3\}$

★ Hole: $(1, \frac{1}{4})$

8) Solve each inequality below. Show the sign graph and write each answer in interval notation.

a) $(2x-3)(3x+4)(x-1) \geq 0$ — sign graph: $-\ +\ -\ +$ at $-\frac{4}{3}, 1, \frac{3}{2}$ — $[-\frac{4}{3}, 1] \cup [\frac{3}{2}, \infty)$

b) $\frac{5}{x+1} \geq \frac{3}{x}$
$\frac{5x}{x(x+1)} - \frac{3x+3}{x(x+1)} \geq 0$
$\frac{2x-3}{x(x+1)} \geq 0$
sign graph: $+\ -\ +$ at $-1, 0, \frac{3}{2}$ — $(-1, 0) \cup [\frac{3}{2}, \infty)$

c) $\frac{(x-3)^2}{x+2} \leq 0$ — sign graph: $-\ +\ +$ at $-2, 3$ — $(-\infty, -2) \cup [3]$

d) $x^2 - 2x < 15$
$x^2 - 2x - 15 < 0$
$(x-5)(x+3) < 0$
sign graph: $+\ -\ +$ at $-3, 5$ — $(-3, 5)$

e) $-3x^2 - x + 2 < 0$
$3x^2 + x - 2 > 0$
$(3x-2)(x+1) > 0$
sign graph: $+\ -\ +$ at $-1, \frac{2}{3}$ — $(-\infty, -1) \cup (\frac{2}{3}, \infty)$

Chapter 3

9) Simplify each of the following:

a) $\frac{x^5 \cdot x^{-7}}{x^{-8}} = \frac{x^{-2}}{x^{-8}} = x^6$

b) $\frac{6x^{\frac{1}{3}} + x^{\frac{-1}{3}}}{x^{\frac{-1}{3}}} = x^{\frac{1}{3}}(6x^{\frac{1}{3}} + x^{-\frac{1}{3}}) = 6x^{\frac{2}{3}} + 1$

c) $(4x^2y)^{-2}(4x^2y)^5 = \frac{4^5 x^{10} y^5}{4^2 x^4 y^2} = 4^3 x^6 y^3 = 64x^6y^3$

d) $3x^0 + (-5x)^0 + (-2x^4y^{-5})^{-3}$

$3 \cdot 1 + 1 + \frac{1}{(-2)^3 x^{12} y^{-15}} = 4 + \frac{y^{15}}{-8x^{12}}$

10) Solve:

a) $2^{5x+2} = 8^{x-4}$ $\quad 2^{5x+2} = (2^3)^{x-4}$

$5x+2 = 3x-12$

$2x = -14$

$x = -7$

b) $3^{x-5} = 4$ $\quad \log_3 4 = x-5$ $\quad \log_3 4 + 5 = x = 6.26$

c) $(x-2)^{\frac{3}{4}} = 64$ $\quad x-2 = 64^{4/3}$

$x-2 = 256$

$x = 258$

d) $16^x = \frac{1}{64}$

$(4^2)^x = 4^{-3}$ $\quad 4^{2x} = 4^{-3}$ $\quad 2x = -3$ $\quad x = -3/2$

e) $(3x)^{\frac{-1}{2}} = 15$ $\quad 3x = 15^{-2}$

$3x = \frac{1}{225}$

$x = \frac{1}{675}$

f) $e^{x+3} = 12$

$\ln 12 = x+3$

$\ln 12 - 3 = x = -.52$

g) $\log_5(x-4) + \log_5(x+4) = 2$

$\log_5(x-4)(x+4) = 2$ $\quad \log_5(x^2-16) = 2$ $\quad 5^2 = x^2 - 16$

$25 + 16 = x^2$

$41 = x^2$

$x = \sqrt{41}$

h) $\log_5(x+5) = 3\log_5 2 + \frac{1}{2}\log_5 9 - \log_5 12$

$\log_5(x+5) = \log_5 2^3 + \log_5 9^{1/2} - \log_5 12$

$\log_5(x+5) = \log_5\left(\frac{8 \cdot 3}{12}\right)$

$\log_5(x+5) = \log_5(2)$

$x+5 = 2$

$x = -3$

i) $\log_4 4^{18} = x-3$

$4^{x-3} = 4^{18}$

$x-3 = 18$

$x = 21$

j) $3^{\log_3 5} = x+13$

$\log_3 x+13 = \log_3 5$

$x+13 = 5$

$x = -8$

11) In 14 days, 52 grams of a certain element decays to 40 grams. What is the half life of this element? Express your answer to 3 decimal places.

$A = P\left(\frac{1}{2}\right)^{t/h} \rightarrow 40 = 52\left(\frac{1}{2}\right)^{14/h} \rightarrow \frac{40}{52} = \left(\frac{1}{2}\right)^{14/h} \rightarrow \left(\frac{40}{52}\right)^{h/14} = \frac{1}{2}$

$\log\left(\frac{40}{52}\right)^{\left(\frac{h}{14}\right)} = \frac{1}{2}$

$14 \cdot \log\left(\frac{40}{52}\right)\left(\frac{1}{2}\right) = h = 36.99$

12) If a $38,000 car depreciates at a rate of 12.5% per year, what is the value of the car in 6 years, to the nearest cent?

$A = P(1-r)^t \rightarrow A = 38000(1-.125)^6 = \$17,054.22$

13) A person invests $45,000 in a certificate of deposit. The investment, which is compounded monthly, grows at a rate of 8.5% each year. How much more is it worth in 6 years, to the nearest cent, if it is compounded continuously?

$A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow A = 45000\left(1+\frac{.085}{12}\right)^{12 \cdot 6} = 74803.50$

$A = P \cdot e^{rt} \rightarrow A = 45000 \cdot e^{.085 \times 6} = 74938.10$

14) If Mr. Smith invests $53,000 in an account paying 7.5% compounded continuously, how long will it take for the money to double? How long will it take to triple?

$A = P \cdot e^{rt}$

$106{,}000 = 53{,}000 \cdot e^{.075t}$

$2 = e^{.075t}$

$\ln 2 = .075t$

$\frac{\ln 2}{.075} = t = 9.24$ years to double

$159{,}000 = 53{,}000 e^{.075t}$

$3 = e^{.075t}$

$\ln 3 = .075t$

$\frac{\ln 3}{.075} = t = 14.65$ years to triple

$74{,}938.10 - 74{,}803.50 = \$134.60$

Chapter 5

Find exact values(no decimals) for problems 1-5 using the following given information:

$\tan\alpha = \frac{8}{-15}$

$\sin\alpha = \frac{8}{17}$ $\cos\alpha = -\frac{15}{17}$ $\sin\beta = -\frac{5}{13}$ $\cos\beta = -\frac{12}{13}$

15) $\sin(\beta-\alpha) = \sin\beta\cdot\cos\alpha - \cos\beta\cdot\sin\alpha = \left(\frac{-5}{13}\right)\left(\frac{15}{17}\right) - \left(\frac{-12}{13}\right)\left(\frac{8}{17}\right) = \frac{-75}{221} - \frac{-96}{221} = \boxed{\frac{21}{221}}$

16) $\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta = \left(\frac{-15}{17}\right)\left(\frac{-12}{13}\right) - \left(\frac{8}{17}\right)\left(\frac{-5}{13}\right) = \frac{180}{221} - \frac{-40}{221} = \boxed{\frac{220}{221}}$

17) $\sin(2\beta) = 2\cdot\sin\beta\cdot\cos\beta = 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right) = \boxed{\frac{120}{169}}$

18) $\tan(2\alpha) = \frac{2\cdot\tan\alpha}{1-\tan^2\alpha} = \frac{2\left(\frac{-8}{15}\right)}{1-\left(\frac{-8}{15}\right)^2} = \frac{\frac{-16}{15}}{1-\frac{64}{225}} = \frac{\frac{-16}{15}}{\frac{161}{225}} = \frac{-16}{15}\cdot\frac{225}{161} = \boxed{\frac{-240}{161}}$

19) $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2 = \frac{225}{289} - \frac{64}{289} = \boxed{\frac{161}{289}}$

Solve for θ, where $0 \le \theta < 360°$. Round to the nearest 10^{th} of a degree.

20) $6\sin^2\theta + \cos\theta - 5 = 0$

$6(1-\cos^2\theta)+\cos\theta-5=0$ → $-6\cos^2\theta+\cos\theta+1=0$ → $(3\cos\theta+1)(2\cos\theta-1)=0$

$6-6\cos^2\theta+\cos\theta-5=0$ → $6\cos^2\theta-\cos\theta-1=0$ → $\cos\theta = -\frac{1}{3}, \frac{1}{2}$ → $\theta = \boxed{109.5°, 250.5°, 60°, 300°}$

21) $4\tan\theta\sin\theta = 3\sin\theta$

$4\tan\theta\sin\theta - 3\sin\theta = 0$ → $\sin\theta(4\tan\theta-3)=0$ → $\sin\theta=0$, $\tan\theta=\frac{3}{4}$

$\boxed{\theta = 0°, 180°, 36.9°, 216.9°}$

Solve for x, where $0 \le x < 2\pi$. Express answers in terms of π radians. Show work.

22) $\csc^2 x = -3\csc x - 2$

$\csc^2 x + 3\csc x + 2 = 0$

$(\csc x + 2)(\csc x + 1) = 0$

$\csc x = -2, -1$

$\sin x = -\frac{1}{2}, -1$ → $x = \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}}$

23) $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\frac{\sqrt{3}}{1}$ → $x = \boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

Chapter 6

Given: $\vec{a} = \langle -3, 4\rangle$, $\vec{b} = \langle -1, 1\rangle$, $\vec{c} = \langle 4, -5\rangle$, $\vec{d} = \langle -2, -3\rangle$

$A(3,-6)$, $B(-2,-4)$, $C(2,-2)$, $D(-5,7)$

Find each of the following:

24) $\|\vec{a}+\vec{b}\| = \boxed{\sqrt{41}}$

$\|\langle -4, 5\rangle\| = \sqrt{(-4)^2+5^2}$

25) $\|3\vec{c}-2\vec{d}\| = \boxed{\sqrt{337}}$

$\|\langle 16, -9\rangle\| = \sqrt{16^2+(-9)^2}$

26) component form of $\overline{AC}$ = $\boxed{\langle -1, 4\rangle}$

$\langle 2-3, -2-{}^-6\rangle$

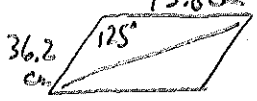
27) component form of $\overline{DB}$ = $\boxed{\langle 3, -11\rangle}$

$\langle -2-{}^-5, -4-7\rangle$

28) Find θ of the resultant of $\vec{c}+\vec{d}$. $\boxed{-76.0°} = \boxed{284.0°}$

$\vec{c}+\vec{d} = \langle 2, -8\rangle$ $\theta = \tan^{-1}\left(\frac{-8}{2}\right) = -76.0°$

29) The sides of a parallelogram are 43.8 cm and 36.2 cm, and the measure of one angle is 125°. Find the area of the parallelogram



$A_\triangle = \frac{1}{2}\cdot b\cdot c\cdot\sin A$

$A_\triangle = \frac{1}{2}(43.8)(36.2)\cdot\sin 125° = 649.41\,cm^2$

$A_\square = 2\cdot A_\triangle = \boxed{1298.81\,cm^2}$

Chapter 9

30) Find the 9th term of an arithmetic sequence if $a_1 = 38$ *and* $a_2 = 30.5$.

$a_n = a_1 + d(n-1)$ / $d = 30.5 - 38 = -7.5$ / $a_9 = 38 - 7.5(9-1) = \boxed{-22}$

31) Find the 12th term of a geometric sequence if $a_1 = 2$ *and* $a_2 = 8$.

$a_n = a_1 \cdot r^{n-1}$ / $r = \frac{8}{2} = 4$ / $a_{12} = 2 \cdot 4^{12-1} = 2 \cdot 4^{11} = \boxed{8{,}388{,}608}$

32) Find the sum of the arithmetic series: 4+12+20+28+36+...+92.

$S_n = \frac{n}{2}(a_1 + a_n)$ / $92 = 4 + 8(n-1) \rightarrow 11 = n-1 \rightarrow 12 = n$ / $S_{12} = \frac{12}{2}(4+92) = 6(96) = \boxed{576}$

33) Find the sum of the geometric series: -729+243-81+...+3.

$S_n = \frac{a_1(1-r^n)}{1-r}$ / $r = -\frac{1}{3}$ / $3 = -729\left(-\frac{1}{3}\right)^{n-1} \rightarrow \frac{3}{-729} = \left(-\frac{1}{3}\right)^{n-1} \rightarrow \left(-\frac{1}{3}\right)^5 = \left(-\frac{1}{3}\right)^{n-1} \rightarrow 5 = n-1 \rightarrow 6 = n$ / $S_6 = \frac{-729(1-(-\frac{1}{3})^6)}{1-(-\frac{1}{3})} = \boxed{-546}$

34) Find the sum of the infinite geometric series: 81+54+36+ ...

$S_\infty = \frac{a_1}{1-r}$ / $r = \frac{54}{81} = \frac{2}{3}$ / $S_\infty = \frac{81}{1-\frac{2}{3}} = \boxed{243}$

35) Write the following series in sigma notation. 21+28+35+....+56.

$d = 7, a_1 = 21$ / $a_n = 21 + 7(n-1)$ / $56 = 21 + 7(n-1) \rightarrow 35 = 7(n-1) \rightarrow 5 = n-1 \rightarrow 6 = n$ / $\boxed{\sum_{n=1}^{6} (21 + 7(n-1))}$

36) Johnny receives birthday gifts for two full weeks (14 days) before and including his birthday. His parents give him a nickel the 1st day, 15 cents the 2nd day, 45 cents the 3rd day, and so on; what will be the total value of this birthday gift?

$.05 + .15 + .45 + \ldots + (.05 \cdot 3^{14-1}) = S_{14} = \frac{.05(1-3^{14})}{1-3} = \boxed{\$119{,}574.20}$

37) Evaluate each of the following:

a) $\sum_{n=1}^{8} (-2)^{n+1}$ $S_8 = \frac{4(1-(-2)^8)}{1-(-2)} = \boxed{-340}$

b) $\sum_{n=1}^{\infty} \left(\frac{-1}{4}\right)^{n-1}$ $S_\infty = \frac{1}{1-(-\frac{1}{4})} = \boxed{\frac{4}{5}}$

c) $\sum_{n=1}^{25} (3n+5)$ $S_{25} = \frac{25}{2}(8+80) = \boxed{1100}$

38) Given: $(2x-3)^{12}$

a) Find the first 4 terms of the expansion.

b) Find the term which has x^3 in its expression.

a) $(2x)^{12} + {}_{12}C_1(2x)^{11}(-3)^1 + {}_{12}C_2(2x)^{10}(-3)^2 + {}_{12}C_3(2x)^9(-3)^3 = \boxed{4096x^{12} - 73{,}728x^{11} + 608{,}256x^{10} - 3{,}041{,}280x^9}$

b) ${}_{12}C_9(2x)^3(-3)^9 = \boxed{-34{,}642{,}080x^3}$

Chapter 10

39) Identify the following first as a circle, ellipse, parabola, or hyperbola. Then, write in Standard form, name all the parts, and sketch the graph.

Hyperbola a) $16y^2 - 9x^2 + 36x + 32y - 164 = 0$

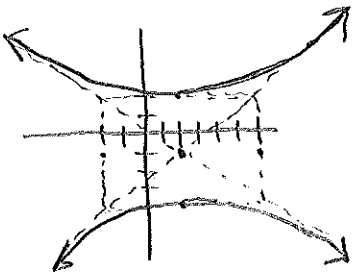
$16y^2 + 32y - 9x^2 + 36x = 164$

$16(y^2+2y+1) - 9(x^2-4x+4) = 164 + 16 + -36$

$16(y+1)^2 - 9(x-2)^2 = 144$

$\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$

C(2,-1)
V(2,2)(2,-4)
F(2,4)(2,-6)



Ellipse b) $4x^2 + 81y^2 + 24x - 486y + 441 = 0$

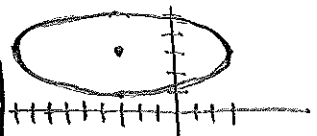
$4x^2 + 24x + 81y^2 - 486y = -441$

$4(x^2+6x+9) + 81(y^2-6y+9) = -441 + 36 + 729$

$4(x+3)^2 + 81(y-3)^2 = 324$

$\frac{(x+3)^2}{81} + \frac{(y-3)^2}{4} = 1$

C(-3,3)
V(6,3)(-9,3)
F(-3+√77,3)(-3-√77,3)



Parabola c) $y^2 + 8x + 4y - 28 = 0$

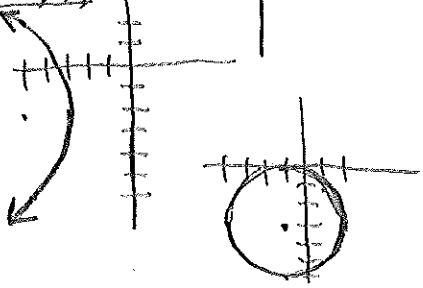
$y^2 + 4y = -8x - 28$

$y^2 + 4y + 4 = -8x - 28 + 4$

$(y+2)^2 = -8x - 24$

$(y+2)^2 = -8(x+3)$

V(-3,-2) axis: y=-2
F(-5,-2) directrix: x=-1



Circle d) $x^2 + y^2 - 2x + 6y + 1 = 0$

$x^2 - 2x + y^2 + 6y = -1$

$(x^2-2x+1) + (y^2+6y+9) = -1 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 9$

$(x-1)^2 + (y+3)^2 = 9$

C(1,-3) r=3

40) Give another pair of polar coordinates for the point $(-4,30°)$ where $r > 0$ and $\theta > 0$.

$(4, 210°)$

41) Change the point (-7,-2) to polar form.

$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right) + 180°$

$r = \sqrt{(-7)^2+(-2)^2} = \sqrt{53} = 7.28$

$\theta = \tan^{-1}\left(\frac{-2}{-7}\right) + 180° = 195.95°$

$(7.28, 195.95°)$

42) Change the point (7,-2) to polar form.

$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$r = \sqrt{7^2+(-2)^2} = \sqrt{53} = 7.28$

$\theta = \tan^{-1}\left(\frac{-2}{7}\right) = -15.95°$

$(7.28, -15.95°)$

43) Change the point $(5,60°)$ to its rectangular form.

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$x = 5 \cdot \cos 60° = 2.5$

$y = 5 \cdot \sin 60° = 4.33$

$(2.5, 4.33)$

44) Change the point $\left(-2, \frac{7\pi}{6}\right)$ to its rectangular form.

$x = r\cos\theta$

$y = r\sin\theta$

$x = -2 \cdot \cos\frac{7\pi}{6} = -2 \cdot \cos 210° = 1.73$

$y = -2 \cdot \sin\frac{7\pi}{6} = -2 \cdot \sin 210° = 1$

$(1.73, 1)$

45) Convert the equations to rectangular form.

a) $r = 9$

$\sqrt{x^2+y^2} = 9$

$x^2+y^2 = 81$

b) $r = \dfrac{2}{1-3\sin\theta}$

$r(1-3\sin\theta) = 2$

$r - 3r\sin\theta = 2$

$r - 3y = 2$

$r = 3y+2$

$\sqrt{x^2+y^2} = 3y+2$

c) $\theta = \dfrac{7\pi}{6}$

$\tan^{-1}\left(\frac{y}{x}\right) = \frac{7\pi}{6}$

$\frac{y}{x} = \tan\frac{7\pi}{6}$

$\frac{y}{x} = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}x$

d) $r = -5\sin\theta$

$r = -5 \cdot \frac{y}{r}$

$r^2 = -5y$

$x^2+y^2 = -5y$

46) Convert the equations to polar form.

a) $9x+4y=2$

$9r\cdot\cos\theta + 4r\cdot\sin\theta = 2$

$r(9\cos\theta + 4\sin\theta) = 2$

$r = \dfrac{2}{9\cos\theta+4\sin\theta}$

b) $x^2+y^2-6y=0$

$r^2 - 6r\sin\theta = 0$

$r^2 = 6r\sin\theta$

$r = 6\sin\theta$

c) $y = -6$

$r\sin\theta = -6$

$r = \dfrac{-6}{\sin\theta}$

$r = -6\csc\theta$

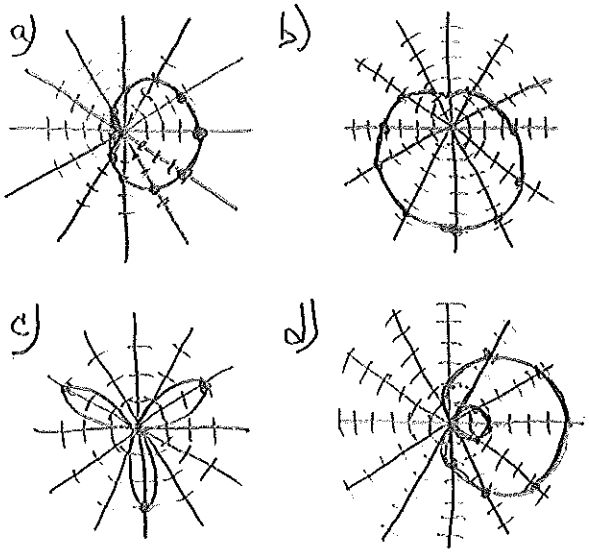
47) Sketch the polar curves below:

a) $r = 2+2\cos\theta$

b) $r = 4-2\sin\theta$

c) $r = 3\sin 3\theta$

d) $r = 2+4\cos\theta$



Chapter 5:

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